HOSSAM GHANEM

(10) 4.4 The Vertical And Horizontal Asymptotes (B)

Example 1
38 March 31, 2004

Let $f(x)=\dfrac{2x^2-x}{(x-3)\sqrt{x^2+1}}$

Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$$f(x)=\frac{2x^2-x}{(x-3)\sqrt{x^2+1}}=\frac{x(2x-1)}{(x-3)\,|x|\sqrt{1+\frac{1}{x^2}}}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x(2x-1)}{x(x-3)\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{2x-1}{(x-3)\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{2-\frac{1}{x}}{\left(1-\frac{3}{x}\right)\sqrt{1+\frac{1}{x^2}}}=2$$

$\therefore\ y=2 \quad H.A$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{x(2x-1)}{-x(x-3)\sqrt{1+\frac{1}{x^2}}}=-2$$

$\therefore y=-2 \quad H.A$

$V.A$

$x-3=0 \quad \rightarrow \quad x=3$

$$\lim_{x\to3^-} f(x)=\lim_{x\to3^-}\frac{x(2x-1)}{(x-3)\sqrt{x^2+1}}=-\infty$$

$$\lim_{x\to3^+} f(x)=\lim_{x\to3^+}\frac{x(2x-1)}{(x-3)\sqrt{x^2+1}}=\infty$$

$\therefore\ x=3 \quad V.A$

Example 2
59 9 July 2011

[2+2 pts.] : Let $f(x)=\dfrac{\sqrt{x^2+1}}{x-3}$

Find all vertical and horizontal asymptotes , if any

Solution

$$f(x)=\frac{\sqrt{x^2+1}}{x-3}=\frac{|x|\sqrt{1+\frac{1}{x^2}}}{x-3}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{|x|\sqrt{1+\frac{1}{x^2}}}{x-3}=\lim_{x\to\infty}\frac{x\sqrt{1+\frac{1}{x^2}}}{x-3}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^2}}}{1-\frac{3}{x}}=1$$

$\therefore\ y=1 \quad H.A$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{|x|\sqrt{1+\frac{1}{x^2}}}{x-3}=-1$$

$\therefore\ y=-1 \quad H.A$

$V.A$

$x-3=0 \quad \rightarrow \quad x=3$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{\sqrt{x^2+1}}{x-3} = -\infty$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{\sqrt{x^2+1}}{x-3} = \infty$$

$$\therefore \ x = 3 \qquad V.A$$

Example 3

53 July 18, 2009 A

Let $f(x) = \dfrac{x\sqrt{3x^2+1}}{x^2-6x+8}$

Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$$f(x) = \frac{x\sqrt{3x^2+1}}{x^2-6x+8} = \frac{x\sqrt{3x^2+1}}{(x-4)(x-2)} = \frac{x\,|x|\sqrt{3+\frac{1}{x^2}}}{(x-4)(x-2)}$$

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x\,|x|\sqrt{3+\frac{1}{x^2}}}{(x-4)(x-2)} = \lim_{x\to\infty} \frac{x^2\sqrt{3+\frac{1}{x^2}}}{x^2-6x+8} = \lim_{x\to\infty} \frac{\sqrt{3+\frac{1}{x^2}}}{1-\frac{6}{x}+\frac{8}{x^2}} = \sqrt{3}$$

$$\therefore \ y = \sqrt{3} \qquad H.A$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{-x^2\sqrt{3+\frac{1}{x^2}}}{x^2-6x+8} = -\sqrt{3}$$

$$\therefore y = -\sqrt{3} \qquad H.A$$

$V.A$

$(x-4)(x-2) = 0 \qquad x = 2 \quad \text{or} \quad x = 4$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x\sqrt{3x^2+1}}{(x-4)(x-2)} = \infty$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x\sqrt{3x^2+1}}{(x-4)(x-2)} = -\infty$$

$$\therefore \ x = 2 \qquad V.A$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \frac{x\sqrt{3x^2+1}}{(x-4)(x-2)} = -\infty$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \frac{x\sqrt{3x^2+1}}{(x-4)(x-2)} = \infty$$

$$\therefore x = 4 \qquad V.A$$

Example 4

50 November 17, 2008 A

Let $f(x) = \dfrac{|x|(x-1)}{x^2+x}$

Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$$f(x) = \frac{|x|(x-1)}{x^2+x} = \frac{|x|(x-1)}{x(x+1)}$$

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{|x|(x-1)}{x^2+x} = \lim_{x\to\infty} \frac{x(x-1)}{x^2+x} = \lim_{x\to\infty} \frac{x^2-x}{x^2+x} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{1-\frac{1}{x}} = 1$$

$$\therefore \ y = 1 \quad H.A$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{|x|(x-1)}{x^2+x} = \lim_{x\to-\infty} \frac{-x(x-1)}{x^2+x} = -1$$

$$\therefore \ y = -1 \quad H.A$$

$V.A$

$$x(x+1) = 0 \qquad \to x = -1 \qquad or \qquad x = 0$$

$$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} \frac{|x|(x-1)}{x^2+x} = \lim_{x\to-1^-} \frac{-x(x-1)}{x(x+1)} = \lim_{x\to-1^-} \frac{-(x-1)}{x+1} = -\infty$$

$$\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} \frac{-(x-1)}{x+1} = \infty$$

$$\therefore \ x = 1 \qquad V.A$$

Example 5 52 April 9, 2009 A	Find the vertical and horizontal asymptotes f, if any , for the graph f (if any) $f(x) = \dfrac{\lvert x-1\rvert}{x^4-x}$

Solution

$$f(x) = \frac{|x-1|}{x^4-x} = \frac{|x-1|}{x(x^3-1)} = \frac{|x-1|}{x(x-1)(x^2+x+1)}$$

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x-1}{x(x-1)(x^2+x+1)} = \lim_{x\to\infty} \frac{1}{x(x^2+x+1)} = 0$$

$$\therefore \ y = 0 \quad H.A$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{-(x-1)}{x(x-1)(x^2+x+1)} = 0$$

$V.A$

$$x(x-1)(x^2+x+1) = 0 \qquad \to \qquad x = 0 \quad \text{or} \quad x = 1$$

$$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} \frac{-(x-1)}{x(x-1)(x^2+x+1)} = \infty$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{-(x-1)}{x(x-1)(x^2+x+1)} = -\infty$$

$$\therefore \ x = 0 \qquad V.A$$

Example 6
49 July 5, 2008

Let $f(x)=\dfrac{3x^2+x|x|}{x^2-x}$

Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$f(x)=\dfrac{3x^2+x|x|}{x^2-x}$

if $x>0$ $\quad f(x)=\dfrac{3x^2+x^2}{x^2-x}=\dfrac{4x^2}{x(x-1)}=\dfrac{4x}{x-1}$

if $x<0$ $\quad f(x)=\dfrac{3x^2-x^2}{x^2-x}=\dfrac{2x^2}{x(x-1)}=\dfrac{2x}{x-1}$

$H.A$

$\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{4x}{x-1}=\lim\limits_{x\to\infty}\dfrac{4}{1-\frac{1}{x}}=4$

$\therefore\ y=4 \quad H.A$

$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{2x}{x-1}=\lim\limits_{x\to-\infty}\dfrac{2}{1-\frac{1}{x}}=2$

$\therefore\ y=2 \quad H.A$

$V.A$

$x-1=0 \quad \rightarrow \quad x=1$

$\lim\limits_{x\to1^-} f(x)=\lim\limits_{x\to1^-}\dfrac{4x}{x-1}=-\infty$

$\lim\limits_{x\to1^+} f(x)=\lim\limits_{x\to1^+}\dfrac{4x}{x-1}=\infty$

$\therefore\ x=1 \quad V.A$

Homework

Find the vertical and horizontal asymptotes of f (if any)

1	$f(x)=\frac{\lvert x\rvert}{2x-3}$	30 October 19, 2000 A
2	$f(x)=\frac{3x\lvert x\rvert}{x^2-1}$	5 April 8, 1993
3	$f(x)=\frac{5x^2+x}{\lvert x\rvert(x-3)}$	14 March 28, 1996
4	$f(x)=\frac{2\sqrt{x^2+7}}{x+7}$	3 March 19, 1992
5	$f(x)=\frac{\sqrt{1-x}}{\lvert x-1\rvert}$	6 April 8, 1993
6	$f(x)=\frac{\sqrt{3x^2+1}}{2-x}$	34 March 23,2002
7	$f(x)=\frac{x-1}{\lvert x+1\rvert}$	54 November 16, 2009 A
8	$f(x)=\frac{\sqrt{x+2}-2}{x+2}$	18 May 24 ,2000
9	$f(x)=\frac{\lvert x-1\rvert(2x+1)}{2x^2+x}$	46 Date: July 5, 2007
10	$f(x)=\frac{3x^2}{\lvert x\rvert(x-1)}$	9 July 2011
11	$f(x)=\frac{2x\sqrt{x^2+1}}{x^2+3x}$	35 October 31, 2002 A
12	$f(x)=\frac{4x^2-1}{(2x+1)\sqrt{4x^2+3}}$	12 November 2, 1995

Homework

Find the vertical and horizontal asymptotes of f (if any)

1	$f(x)=\frac{1}{x-x^2}$	5	$f(x)=\frac{7x}{\sqrt{5x^2+7}}$
2	$f(x)=\frac{x^2+1}{x^2-1}$	6	$f(x)=\frac{1}{x^2-1}+2$
3	$f(x)=\frac{x-2}{x^2}$	7	$f(x)=\frac{3\lvert x-1\rvert}{x-x^3}$
4	$f(x)=\frac{x^3-16}{x}$	8	$f(x)=\frac{3x^2}{\lvert x\rvert(x-1)}$

11 35 October 31, 2002 A	Find the vertical and horizontal asymptotes f, if any , for the graph f (if any) $f(x)=\frac{2x\sqrt{x^2+1}}{x^2+3x}$

Solution

$$f(x)=\frac{2x\sqrt{x^2+1}}{x^2+3x}=\frac{2x\sqrt{x^2+1}}{x(x+3)}=\frac{2x\,|x|\sqrt{1+\frac{1}{x^2}}}{x(x+3)}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{2x^2\sqrt{1+\frac{1}{x^2}}}{x^2+3x}=\lim_{x\to\infty}\frac{2\sqrt{1+\frac{1}{x^2}}}{1+\frac{3}{x}}=2$$

$\therefore\ y=2 \quad H.A$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{-2x^2\sqrt{1+\frac{1}{x^2}}}{x^2+3x}=-2$$

$\therefore y=-2 \quad H.A$

$V.A$

$x(x+3)=0 \quad \rightarrow \quad x=-3 \quad \text{or} \quad x=0$

$$\lim_{x\to-3^-} f(x)=\lim_{x\to-3^-}\frac{2x\sqrt{x^2+1}}{x(x+3)}=\lim_{x\to-3^-}\frac{2\sqrt{x^2+1}}{x+3}=-\infty$$

$$\lim_{x\to-3^+} f(x)=\lim_{x\to-3^+}\frac{2\sqrt{x^2+1}}{x+3}=\infty$$

$\therefore\ x=-3 \quad V.A$